PHYSICAL REVIEW E **69**, 065402(R) (2004)

Alfvén limit in fast ignition

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(Received 19 December 2003; revised manuscript received 4 March 2004; published 17 June 2004)

Fast ignition inertial confinement fusion relies on rapidly heating the compressed fuel to ignition using a laser-generated electron beam. The current required far exceeds the Alfvén limit, so it can only propagate while the plasma provides a nearly coincident return current. The resistive decay of the return current is shown to be too rapid for the originally proposed scheme to work. Possible solutions to this problem are to increase the mean energy of the beam, to heat the fuel to a higher temperature by lowering the beam radius and duration, to use multiple beams, and to use an annular beam. Considering the laser wavelength required shows that increasing the mean energy and number of beams are the most practical solutions.

DOI: 10.1103/PhysRevE.69.065402 PACS number(s): 52.57.Kk, 52.59.−f, 41.75.−i

The fast ignition scheme for inertial confinement fusion proposed by Tabak *et al.* [1] relies on rapidly heating the core of the compressed fuel to ignition using a laser-generated electron beam. This has advantages over the conventional scheme of a spherically converging shock wave in that it could achieve the higher gain of the isochoric model over the isobaric model [2], that it does not require a high degree of symmetry, and that it could have a higher efficiency. Alfvén [3] noted that the main limiting factor on the propagation of an electron beam in a conductor is the self-generated magnetic field, which acts to turn the electrons back towards the source, limiting the current to a value of the order of

$$J_A = \frac{4\pi}{e\mu_0} p, \tag{1}$$

known as the Alfvén limit, where e is the electron charge and p is the electron momentum. The current required for fast ignition is given by

$$J_{FI} = \frac{eK_t}{\langle K\rangle t_b}, \tag{2}$$

where K_t is the total electron energy, $\langle K\rangle$ is the mean electron energy, and t_b is the beam duration. Fast ignition requires a minimum energy for ignition, a maximum mean fast electron energy, to ensure that the electrons stop in the core, and a maximum beam duration, to avoid expansion of the core, therefore it requires a minimum current. Tabak *et al.* estimated these parameters to be 3 kJ, 1 MeV, and 10 ps, respectively, for a deuterium-tritium (DT) fuel density of 3×10^5 kg m^{-3} (300 g cm^{-3}), giving a minimum ignition current of 0.3 GA. More detailed calculations by Atzeni [2] put the ignition energy at 18 kJ and the pulse duration at 20 ps, increasing this value to 0.9 GA. The Alfvén limit [Eq. (1)] for 1 MeV electrons is 47.5 kA, a factor of 1.9×10^4 lower than this, indicating that the scheme is unworkable. However, this is a limit on the net current, not the forward current, and as the electron beam enters the plasma the initial charge separation will draw an equal, almost coincident, return current from the plasma, giving a net current very much lower than the forward current. However, the return current is in turn opposed by the effect of collisions and separates from the beam current due to their mutual repulsion, giving a net current that increases in time. This means that a current that exceeds the Alfvén limit can only propagate for a limited time. In this Rapid Communication an upper limit for this magnetic inhibition time is derived, which corrects a previous derivation [4], and it is used to reconsider the beam requirements for fast ignition.

The simplest model for the plasma return current is the basic Ohm's law $\mathbf{E}=\eta\mathbf{j}_p$, where $\mathbf{E}$ is the electric field, $\mathbf{j}_p$ is the plasma current density, and η is the plasma resistivity. The use of this equation in the context of laser-generated electron transport is discussed in some detail by Glinsky [5]. It assumes that the dynamics of the plasma electrons is dominated by collisions. This is an adequate approximation provided that the plasma electron density is much greater than that of the beam. This is certainly the case in the compressed fuel, but it may not be the case near the region where the electrons are generated, which will be at the laser critical density. As the beam current is much higher than the limiting value the beam current density $\mathbf{j}$ must be almost exactly balanced by the plasma current density, so we can use $\mathbf{j}_p\approx-\mathbf{j}$. Substituting the resulting expression for the electric field into Faraday's law gives the growth rate of the magnetic field as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \eta\mathbf{j}. \tag{3}$$

Assuming a constant resistivity and a constant current density, we can estimate the magnetic field from Eq. (3) to be $B\sim\eta jt/R$, where R is the beam radius for a cylindrical beam. The net current J_{net} is $2\pi RB/\mu_0$ and the beam current J is $\pi R^2 j$, giving

$$J_{\mathrm{net}} \sim 2\frac{t}{t_D}J, \tag{4}$$

where

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$$t_D = \frac{\mu_0 R^2}{\eta} \quad (5)$$

is normally referred to as the magnetic diffusion time, but as diffusion of the magnetic field has not been included here, due to the assumption of approximate current balance, I will refer to it as the current decay time, to avoid confusion. The assumption of approximate current balance is only valid for times much less than this, as is obvious from Eq. (4), which predicts a net current greater than the beam current for $t > t_D/2$. If magnetic diffusion is included, the net current tends to the beam current. The Alfvén limit thus applies for times greater than the current decay time. This is much lower than the times relevant to the propagation of cosmic rays, the application Alfvén considered, and to most cases of interest in beam physics, but it is typically greater than the duration of laser-generated electron beams, so we must consider the time dependent problem. This calculation only takes into account the resistive decay of the plasma return current, the mutual repulsion between the beam and plasma currents has not been taken into account, so Eq. (5) gives an upper limit on the current decay time. It is thus an adequate model for determining if magnetic inhibition will be important. In practice, the effect of the mutual repulsion on the plasma electrons is only likely to be significant near the region where the electrons are generated, as the beam and plasma densities could be comparable. The effect on the beam electrons is to cause pinching, otherwise known as magnetic focusing, which does not affect these calculations because it is the final beam radius that is of interest. It only affects the laser conditions required to achieve it. The time at which the net current given by Eq. (4) exceeds the Alfvén limit gives the magnetic inhibition time

$$t_I \sim \frac{t_D}{2J/J_A}. \quad (6)$$

The maximum radius given by Atzeni's ignition power and intensity thresholds is 20 μm, and the Spitzer resistivity at the final, ignition temperature (kT/e) of approximately 10 keV is of the order of 1 nΩ m, giving a current decay time [Eq. (5)] of 50.3 μs, which is much greater than the beam duration, justifying the assumption of current balance. We estimated J/J_A to be at least 1.9×10^4, giving $t_I/t_b \sim 0.65$, so the minimum ignition current cannot be maintained for the required time. The parameters originally suggested by Tabak *et al.*, for which the radius is 10 μm, are actually worse in this respect, giving $t_I/t_b \sim 0.24$.

The reader may not be convinced by such a crude treatment, so I will now give a more rigorous one. For this I will use the Bennet current profile [6]

$$j_B = \frac{j_0}{(1 + r^2/R^2)^2}, \quad (7)$$

which has a total current J given by $\pi R^2 j_0$. This is chosen above all for of its mathematical simplicity. Its most important feature is not that it is an equilibrium solution, but that it peaks on-axis and falls with radius, features that would be expected of a "typical" beam. I will discuss the effect of varying the current profile later. The magnetic field [Eq. (3)] for a constant resistivity and a constant current density given by Eq. (7) is

$$B = \frac{4r/R}{(1 + r^2/R^2)^3} \frac{\eta j_0 t}{R}. \quad (8)$$

This gives a net current density from $\nabla \times \mathbf{B}/\mu_0$ of

$$j_{\rm net} = 8 \frac{t}{t_D} \frac{1 - 2r^2/R^2}{(1 + r^2/R^2)^4} j_0. \quad (9)$$

Thus there is a net forward current up to a radius of $R/\sqrt{2}$, and a net return current at larger radii. The net forward current is

$$J_{\rm net} = \frac{32}{27} \frac{t}{t_D} J, \quad (10)$$

which is close to the simple estimate of Eq. (4). An absolute upper limit on the current can be obtained by calculating when the energy per unit length in the magnetic field would equal that of the particles which generated it [4]. The latter is given by $J_{\rm net}\langle K\rangle/ev$, where $\langle K\rangle$ is the mean forward energy of the electrons and v is the propagation velocity of the beam. To determine the absolute upper limit we should assume that all of the electrons are traveling forwards, so that $\langle K\rangle$ is the mean electron energy and v the mean electron velocity. This equals the energy per unit length in the magnetic field given by Eq. (8) when

$$t_I = \frac{20}{27} \frac{4\pi}{e\mu_0} \frac{\langle K\rangle}{v} \frac{1}{J} t_D. \quad (11)$$

This differs from the result of Ref. [4] because there the total energy in the forward current was used, rather than that in the net forward current, which is the energy actually available to generate the magnetic field. That this is the correct approach is clear when we consider the limit as being that for the current density given by Eq. (9), rather than that for the Bennet profile [Eq. (7)]. For a strongly relativistic, monoenergetic beam $\langle K\rangle/v \approx p$, so Eq. (11) can be written as $t_I \approx t_D/(1.35J/J_A)$, which is only a factor of about 1.5 greater than the simple estimate of Eq. (6). Repeating these calculations for a Gaussian current profile $j_0 \exp(-r^2/R^2)$ gives a magnetic inhibition time a factor of $\exp(0.5)/1.35 \approx 1.22$ lower than Eq. (11). This is a result of the sharper fall off of the Gaussian profile, which contains $1-\exp(-1) \approx 0.632$ of its total current within the radius R, compared to 0.5 for the Bennet profile. The ratio of the inhibition time [Eq. (11)] to the beam duration is given by

$$f_p \equiv \frac{t_I}{t_b} \approx 3.10 \times 10^{-8} \frac{c}{v} \left(\frac{\langle K\rangle}{e} \right)^2 \frac{R^2}{K_t} \frac{1}{\eta}, \quad (12)$$

which I will refer to as the propagation factor. For Atzeni's ignition parameters $f_p \approx 0.73$, which confirms the previous estimate.

Thus we can conclude that only a fraction of the beam will enter the compressed fuel before the magnetic field grows sufficiently to start turning electrons back towards the source of the beam. We would expect the majority of these electrons to spread out in the corona, rather than return to the core. Such behavior has been seen in particle-in-cell modeling of similar situations [7]. Numerical solutions of a model similar to that used here have shown that energy deposition is considerably reduced by magnetic inhibition if electrons returned to the source are removed, and concentrated near the source if they are reflected [8]. Therefore more energy must be put into the beam, but this increases the current, lowering the magnetic inhibition time. Clearly we require the propagation factor to be greater than one for an ignition beam. Before considering how it could be increased, we should consider by how much it must, realistically, be increased by, because in obtaining the value of 0.73 we were looking to obtain an upper limit, so that we could be certain that magnetic inhibition is important. To be certain that it will not be important we must take the opposite approach. The magnetic field turns electrons back before they lose all of their forward current in generating it [4], so we can count on an actual propagation factor half that of Eq. (12). The Bennet profile contains only half the total current within the radius R, and for more sharply peaked profiles the inhibition time is lower, so we can count on at least another factor of 2. The total electron energy used was the energy that must be uniformly deposited in a cylindrical region of the core, the actual value must be higher than this to take account of loses in reaching the core, of scattering of electrons out of the region, and of the wide energy spectra of laser-generated electron beams, which contain electrons with energies too high and too low to produce useful heating. We can thus count on at least doubling the total energy. The use of a lower value for the resistivity may appear to be unreasonable, but it has been shown that the final resistivity does give a good estimate of the final magnetic field when there is a large increase in the temperature [9]. Taking all of these factors into account, we see that, to be on the safe side, we should consider a propagation factor four times lower than that of Eq. (12) and a total energy of 36 kJ, so we need to increase it 11 times. The propagation factor is independent of the beam duration. It might also be expected to be independent of the fuel density ρ, but the density scaling of Atzeni's ignition thresholds gives $K_t/R^2 \propto \rho^{0.15}$ and hence $f_p \propto \rho^{-0.15}$. He attributes this departure from the expected scaling to the fall in $\ln \Lambda$ with density. As this also appears in the electron stopping power the maximum mean electron energy will also fall with density, further lowering the propagation factor. In spite of this, we would still expect the density scaling of the propagation factor to be relatively insignificant. In order to achieve ignition we must have a minimum energy per area K_t/R^2, so the propagation factor can be increased by increasing the mean energy and by lowering the resistivity. Increasing the mean energy increases the distance over which the energy is deposited. Atzeni found the ignition thresholds to be weakly affected by changing this distance by a factor of 2 either way, but he only considered distances much less than the diameter of the core. I will assume that this distance can be doubled with no increase in the total energy, but that all energy deposited beyond this distance is wasted. The collisional stopping distance s of an electron varies approximately as $s \propto K^2/\gamma$, where K is the electron's kinetic energy and γ is the Lorentz factor, given by $1+K/mc^2$. Assuming that the distance over which the energy is deposited is given by the stopping distance at the mean electron energy, and using the strongly relativistic approximation $s \propto \langle K \rangle$, we obtain $f_p \propto \langle K \rangle$ and $K_t \propto \langle K \rangle$, for increases in the mean energy above 2 MeV, so the desired increase could be achieved by increasing the mean energy to 5.5 MeV and the total energy to 99 kJ. The resistivity can be lowered by increasing the temperature T to which the fuel is heated, as the Spitzer resistivity $\eta \propto T^{-3/2}$. Ignoring thermal conduction and alpha particle heating we have $T \propto K_t R^{-2} s^{-1}$, where s is again taken to be the collisional stopping distance at the mean electron energy. The temperature can thus be increased by increasing the total energy, reducing the radius, and reducing the mean energy. For the total energy we have $f_p \propto K_t^{1/2}$, making this is a very inefficient solution. For the radius we have $f_p \propto 1/R$, so the desired increase could be achieved by reducing the radius to 1.8 μm. For the mean energy we have $f_p \propto \langle K \rangle^{-3/2} \gamma^{5/2} (1+\gamma)^{-1/2}$, so the desired increase could be achieved by reducing the mean energy to 50 keV, but at the same time the temperature (kT/e) is increased to 1.4 MeV, so this is not possible. In reducing the radius to 1.8 μm the temperature is increased to 1.2 MeV, so this must be combined with an increase in the mean energy. The temperature could also be increased by reducing the beam duration, as this reduces cooling due to thermal conduction. Such a reduction is implicit in all of the above proposals. For Atzeni's parameters, if the energy were to be deposited instantaneously the temperature would be approximately doubled, leading to an increase of approximately 2.8 times in the propagation factor, well short of that required. The problem could also be overcome by making more fundamental changes to the scheme itself. The most obvious of these is to use multiple beams, each containing a fraction of the total energy. As a result of the limited overlap possible between the beams, we can count on increasing the total energy required by a factor of about 4 [10], so 44 beams with a total energy of 144 kJ would be required to achieve the desired increase by this means alone. As each beam has a radius of 20 μm they would fill practically the whole surface area of the core, which suggests the use of spherically symmetric irradiation. This would avoid alignment problems inherent in this scheme, remove the inefficiencies of overlapping beams and, in theory, remove the magnetic field. In practice, inevitable irregularities in the irradiation and in the compressed fuel will lead to some magnetic field generation. This is similar to the conventional scheme, but using a spherically converging heat wave instead of a shock wave. It also has similarities to the coronal ignition scheme of Hain and Mulser [11], in which ignition is triggered by a heat wave initiated at the laser critical density. Another possibility is to use a hollow electron beam to increase the current limit [4]. The Alfvén limit for an annular beam is greater than that of Eq. (1) by a factor of approximately the radius of the annulus R divided by its width ΔR. As only a narrow annulus would be heated thermal conduction and alpha particle heating would have to be relied upon to heat the central region. Heat flow out of this region would make the scheme less efficient, so to be on the safe side I will assume that total energy has to be doubled to 72 kJ. To give an Alfvén limit higher than the beam current [Eq. (2)], would require $R/\Delta R \approx 7.6\times10^4$, giving $\Delta R=0.26$ nm for $R=20$ μm. However, the propagation factor could be increased sufficiently for a lower value of $R/\Delta R$ because the temperature is increased, lowering the resistivity. Neglecting thermal conduction, $T\propto R/\Delta R$, which for the Spitzer resistivity $\eta \propto T^{-3/2}$ gives $f_p \propto (R/\Delta R)^{1/2}$, as it is ΔR that determines the current decay time [Eq. (5)], not R, so $\Delta R=41$ nm could be sufficient. As this requires a temperature of 2.4 MeV the mean energy would have to be increased. The beam duration would also have to be reduced to prevent cooling. As a means of lowering the resistivity this is less efficient than simply lowering the radius.

An important factor to be taken into account when deciding which solution to choose is the maximum laser wavelength λ required, which results from the requirements for a minimum laser intensity I and a maximum electron energy, which puts a maximum value on $I\lambda^2$. A low wavelength is undesirable because it brings problems with laser efficiency and focusing optics. The laser intensity required for fast ignition is

$$I_{FI} = \frac{K_t}{f_{abs}\pi R^2 t_b}, \tag{13}$$

where f_{abs} is the fraction of the laser energy converted to beam energy. The radius R in Eq. (13) could be greater than the beam radius due to magnetic pinching of the beam, however, analytical and numerical results indicate that this will be strongly limited by the fall in resistivity resulting from the large increase in temperature [9,12], so I will take it to be the beam radius. The mean electron energy at relativistic intensities has been found to be approximately given by the ponderomotive potential, or maximum oscillation energy of an electron in the laser field. In the strongly relativistic limit, $I\lambda^2 \gg 10^{10}$ W, this is approximately $4.77(I\lambda^2)^{1/2}$ eV, giving

$$\lambda_{FI} \approx 0.37\frac{\langle K\rangle R f_{abs}^{1/2} t_b^{1/2}}{eK_t^{1/2}}. \tag{14}$$

For $\langle K\rangle/e=1$ MeV, $R=20$ μm, $f_{abs}=0.5$, which is about the highest value that can be reasonably expected, $t_b=20$ ps, and $K_t=36$ kJ, we have $\lambda_{FI}\approx 120$ nm. Reducing this would be undesirable, so the methods of choice for increasing the propagation factor are increasing the mean energy and the number of beams, as these increase the maximum wavelength, whereas the other methods reduce it considerably. The proposed increases in the mean energy and number of beams both give $\lambda_{FI}\approx 0.40$ μm, a perfectly practical value. It should also be taken into account that magnetic inhibition could still occur due to the mutual repulsion between the beam and return currents. This requires a different treatment, but it is obvious that lowering the beam current will also reduce this effect. This again favors increases in the mean energy and the number of beams.

In conclusion, it has been shown that the resistive decay of the return current will prevent the propagation of the electron beam envisaged for fast ignition. Increasing the mean energy, increasing the temperature by lowering the beam radius and duration, using multiple beams, and using an annular beam have been shown to be possible solutions to this problem. Taking into account the laser wavelength required shows that increasing the mean energy and number of beams are the most practical solutions. In the terminology used by Atzeni, these considerations reduce the "ignition window," requiring the ideal conditions to be reconsidered.

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